

# The uncertainty in a current meter measurement

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## Abstract

All measurements of physical quantities are subject to uncertainties. These may be due to bias errors in the equipment used for calibration and measurement, or to random scatter caused by, for example, a lack of sensitivity of the equipment used for the measurement [1]. Every day, throughout the world, numerous current meter measurements are made in open channels to measure flow without any report on the uncertainty of the measurement.. The uses made of these measurements in the design and operation of river works and in water resources management generally, require an assessment of the reliability of the measurements and in such cases it is important that the uncertainty of the measurements is reported [1]. New international recommendations involve the analysis of Type A and Type B methods of evaluation of uncertainty, the result to be reported as a combined uncertainty with symbol  $u_c$  and a coverage factor of 2 corresponding to a confidence level of approximately 95% [2].

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## 1. Introduction

The result of a measurement is only an estimate of the true value of the measurement and is therefore only complete when accompanied by a statement of its uncertainty.

The discrepancy between the true and measured values is the measurement error. The measurement error, which cannot be known, causes an uncertainty about the correctness of the measurement result.

The measurement error is a combination of component errors that arise during the performance of various

elementary operations during the measurement process. For measurements of composite quantities, that depend on several component quantities, the total error of the measurement is a combination of the errors in all component quantities. Determination of measurement uncertainty involves identification and characterization of all components of error and the quantification and combination of the corresponding uncertainties [3].

## Nomenclature

$u_{pi}$	uncertainty in mean velocity $v_i$ due to the limited number of depths at which velocity measurements are made at vertical $i$
$n_i$	number of depths in the vertical at which velocity measurements are made
$u_{ci}$	uncertainty in point velocity at a particular depth in vertical $i$ due to variable responsiveness of the current meter
$u_{ei}$	uncertainty in point velocity at a particular depth in vertical $i$ due to velocity fluctuations (pulsations) in the stream

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## 2. Analysis of uncertainties under the new international recommendations

The analysis of uncertainties in flow measurement now follows the procedure recommended in the International Standards Organization's *Guide to the Expression of Uncertainty in Measurement* [2]. The Guide was developed by seven international organizations and published by ISO. The procedure adopted with respect to the uncertainty analysis of current meter flow data differs from previous methodology used in the past in both terminology and presentation [4,5]. Previously, the influences that gave rise to uncertainty were recognized as 'random' or 'systematic' and reported separately and combined. The concept of the Guide is that there is no inherent difference between the uncertainty component arising from a random effect and one arising from a correction for a systematic effect. Indeed, 'random' and 'systematic' uncertainties no longer enter into uncertainty analysis. The international community have agreed that the procedure laid down by the Guide should be followed and all existing international and national standards should conform accordingly.

The components of uncertainty are characterized by estimates of standard deviation, that are termed standard uncertainty, with the recommended symbol  $u_i$  and which are equal to the positive square root of the estimated variance  $u_i^2$ . The uncertainty components are combined using equations for the combination of standard deviations. The resultant uncertainty, which takes all sources and components of uncertainty into account, is now defined as the combined uncertainty.

## 3. Type A and Type B evaluation of uncertainties

The Guide introduces the concept of Type A and Type B methods of evaluation of uncertainty to make a distinction between uncertainty evaluation by statistical analysis of replicate measurements and uncertainty evaluation by other (perhaps subjective or judgment) means. Type A evaluation of uncertainty is by statistical analysis of repeated observations to obtain statistical estimates of the standard deviation of the observations. Type B evaluation of uncertainty is by calculation of the standard deviation of an assumed probability distribution based on scientific judgement and consideration of all available information that may include previous measurement and calibration data and experience or general knowledge of the behaviour and properties of relevant instruments. By proper consideration of correlations, either Type A or Type B method of evaluation of uncertainty can be used for evaluation of either random or systematic uncertainty components.

According to the Guide, all uncertainties in flow are expressed as percentage standard uncertainties corre-

sponding to percentage coefficients of variation (standard deviation divided by the mean). Expanded uncertainties are explicitly identified as such and are taken with coverage factor 2, corresponding to a confidence level of approximately 95% [2]. That is, 95% of the observations should, on average, be within the specified limits of two standard deviations from the mean. It should be noted, however, that no uncertainty value can be ascribed to the remaining 5% [1].

## 4. Procedure to calculate the uncertainty in a current meter flow measurement

The new recommendations in the Guide for estimating the uncertainty in a current meter gauging will be demonstrated by the following typical example [5].

The measurement method, briefly, consists of dividing the channel cross-section under consideration into segments by  $m$  verticals and measuring the breadth, depth and mean velocity (denoted by  $b_i$ ,  $d_i$ ,  $v_i$  respectively) associated with each vertical  $i$ . The mean velocity  $v_i$  at each vertical is computed from point velocity measurements made at each of several depths on the vertical. The flow is calculated as follows [6]:

$$Q = F \sum b_i d_i v_i \quad (1)$$

Where  $Q$  is the flow (in  $\text{m}^3\text{s}^{-1}$ ) and  $F$  is a factor, assumed to be unity, that relates the discrete sum over the finite number of verticals to the integral of the continuous function over the cross-section [4]. That is, Eq. (1) requires to be optimised until sufficient verticals are employed so as to make  $F$  unity. If this is not the case  $F$  may be greater than unity since discharges calculated from river sections having too few verticals are generally too low.

The relative (percentage) combined standard uncertainty in the measurement is given by the following equation:

$$u(Q)^2 = u_m^2 + u_s^2 + \frac{\sum ((b_i d_i v_i)^2 (u_{b_i}^2 + u_{d_i}^2 + u_{v_i}^2))}{(\sum b_i d_i v_i)^2} \quad (2)$$

where  $u(Q)$  is the relative (percentage) combined standard uncertainty in discharge,  $u_{b_i}$ ,  $u_{d_i}$ ,  $u_{v_i}$  are the relative (percentage) standard uncertainties in the breadth, depth, and mean velocity measured at vertical  $i$ .

$u_s$ =uncertainty due to calibration errors in the current meter, breadth measurement instrument, and depth sounding instrument:

$=(u_{cm}^2 + u_{bm}^2 + u_{ds}^2)^{1/2}$ . An estimated practical value of 1% may be taken for this expression.

$u_m$ =uncertainty due to the limited number of verticals  
 $m$ =number of verticals

Now,

$$u(v_i)^2 = u_{pi}^2 + (1/n_i)(u_{ci}^2 + u_{ei}^2) \quad (3)$$

Combining Eqs. (2) and (3) yields:

$$u(Q)^2 = u_m^2 + u_s^2 + \frac{\sum ((b_i d_i v_i)^2 (u_{bi}^2 + u_{di}^2 + u_{pi}^2 + (1/n_i)(u_{ci}^2 + u_{ei}^2)))}{(\sum b_i d_i v_i)^2} \quad (4)$$

If the measurement verticals are placed so that the segment discharges ( $b_i d_i v_i$ ) are approximately equal and if the component uncertainties are equal from vertical to vertical, then Eq. (4) simplifies to:

$$u(Q) = [u_m^2 + u_s^2 + (1/m)(u_b^2 + u_d^2 + u_p^2 + (1/n)(u_c^2 + u_e^2))]^{1/2} \quad (5)$$

## 5. Typical example of a Type B evaluation of uncertainty

It is required to calculate the uncertainty in a current meter gauging from the following particulars [5, 7]:

Number of verticals used in the gauging: 20  
Exposure time of current meter at each of 2 points: 3 min  
Number of points taken in the vertical (0.2 and 0.8): 2  
Average velocity in measuring section: above 0.3 m/s  
Rating of current meter: individual rating

The following typical percentage component uncertainty values are taken from ISO 748 [4]:

$u_m$  2.5  
 $u_s$  1.0 (see above)  
 $u_b$  0.5  
 $u_d$  0.5  
 $u_p$  3.5  
 $u_c$  1.0  
 $u_e$  2.5 (at 0.2 depth)  
2.5 (at 0.8 depth)

Therefore, from Eq. (5):

$$\begin{aligned} u(Q) &= [u_m^2 + u_s^2 + (1/m)(u_b^2 + u_d^2 + u_p^2 + (1/n)(u_c^2 + u_e^2))]^{1/2} \\ &= [2.5^2 + 1^2 + (1/20)(0.5^2 + 0.5^2 + 3.5^2 + (1/2)(1.0^2 + 2.5^2))]^{1/2} \\ &\text{giving } u(Q) = 2.84\%, \text{ say } 3\% \\ &\text{Expanded uncertainty, } U, \text{ coverage factor } k = 2, \\ &\text{approximate confidence level } 95\% \end{aligned}$$

Therefore

$$U(k = 2)(Q) = ku(Q) = 2 \times 3 = 6\%$$

Therefore

$$U(Q) = 6\%$$

Now, if the measured flow is  $Q \text{ m}^3/\text{s}$ , the result of the measurement is expressed as:  $Q \text{ m}^3/\text{s} \pm 6\%$  (expanded uncertainty, coverage factor  $k = 2$ , approximate level of confidence = 95%).

Note: The above uncertainty calculation is considered to be a Type B evaluation of uncertainty since the component uncertainties in ISO 748 are based on previous measurements, calibration data and research carried out over the last forty years [8–28].

## 6. Conclusions

The new international method of estimating the uncertainty in measurement as recommended in the *Guide to the Expression of Uncertainty in Measurement* [2] has now been adopted worldwide for physical measurements. The procedure is based on sound principles of mathematical statistics and is simple to apply to open channel flow measurement. In applying the concept to open channel measurement, or to hydrometry in general, it will normally be a Type B evaluation that is considered for general current meter gauging but for research or special studies, a Type A evaluation will normally be considered.

## References

- [1] R.W. Herschy, *Hydrometry Principles and Practices*, second ed., Wiley, Chichester, UK, 1998.
- [2] *Guide to the expression of uncertainty in measurement*, ISO, Geneva, 1995.
- [3] W. Kirby, USGS. Private communication, 2001–2002.
- [4] ISO 748, *Velocity area methods*, International Standards Organization, Geneva, 1997.
- [5] ISO 1088, *Collection of data for determination of errors in measurement by velocity area methods*, International Standards Organization, Geneva, 1973. (under revision).
- [6] R.W. Herschy, *Streamflow Measurement*, second ed., Spon, UK, 1995.
- [7] ISO 5168, *Calculation of the uncertainty of a measurement of flow-rate*, International Standards Organization, Geneva, 1978. (under revision).
- [8] Dementev, V.V., 1962. Investigation of pulsations of velocities of flow of mountain streams and its effect on the accuracy of discharge measurements. Translations from Soviet Hydrology by D.B. Krimgold and published by the American Geophysical Union, No. 6, 558–623.
- [9] Carter, R.W., Anderson, I.E., 1963. Accuracy of current meter measurements. *Proc. Am. Soc. Civ. Eng.*, 89 (HY4) 105–115.
- [10] Smoot, G.F., Carter, R.W., 1968. Are individual current meter ratings necessary? *Proc. Am. Soc. Civ. Eng.* 94 No. 5848.
- [11] Herschy, R.W., 1969. The magnitude of errors at flow measurement stations, *Water Resources Board*, T.N. No. 5, 30.
- [12] Botma, H., Struijk, A.J., 1970. Errors in measurement of flow by velocity area methods. *Proc. International Symposium on*

- Hydrometry, Koblenz, UNESCO, WMO, IAHS, Pub. No. 99, 86–98.
- [13] Herschy, R.W. The magnitude of errors at flow measurement stations. Proc. International Symposium on Hydrometry, Koblenz, UNESCO, WMO, IAHS, Pub. No. 99 1970, 109–126.
  - [14] Grindley, J. The calibration and behaviour of current meters:
    - HRS Report INT 80, 1970. Methods of calibration
    - HRS Report INT 87, 1971. Effect of oblique flow
    - HRS Report INT 93, 1971. Effect of suspension
    - HRS Report INT 95, 1971. Accuracy
    - HRS Report INT 96, 1971. Drag
    - HRS Report INT 99, 1972. Tests in flowing water
  - [15] Herschy, R.W., The accuracy of existing and new methods of river gauging. PhD thesis, The University of Reading, 1975, 495.
  - [16] Herschy, R.W., Hindley, D.R., Johnson, D., Tattersall, K.H. The effect of pulsations on the accuracy of river flow measurement, Tech. Mem. No.10 Water Data Unit, Reading, UK, 1978, 48.
  - [17] Herschy, R.W. The accuracy of current meter measurements. Proc. Inst. Civ. Engrs., Part 2, 65, 1978, 431–437.
  - [18] Herschy, R.W. An examination of the variation of mean monthly discharge of British rivers from a short period of record to assess the needs for planning and design. Proc. Instn. Civ. Engrs., Part 1, 68, 1980, 477–488.
  - [19] Herschy, R.W. Current meter calibration: Individual rating versus group rating, IAHS Pub. No. 134, 1982, 25–36.
  - [20] J.R. Dymond, R. Christian, Accuracy of discharge determined from a rating curve, Hydrological Science Journal 27 (4) (1982) 493–504.
  - [21] R.W. Herschy, Accuracy, in *Hydrometry Principles and Practices*, Wiley, Chichester, UK, 1978.
  - [22] R.W. Herschy, Accuracy, in *Streamflow Measurement*, Elsevier Applied Sciences Pub, UK, 1985.
  - [23] Pelletier, P.M., Uncertainty in the single determination of river discharge: a literature review, CSCE Centennial Conference Montreal, 1987.
  - [24] M. Lintrup, A new expression for the uncertainty of a current meter discharge measurement, Nordic Hydrology 20 (1989) 191–200.
  - [25] ISO 7066/1, Uncertainty in linear calibration relations, International Standards Organization, Geneva, 1989.
  - [26] Sauer, V.B., Meyer, R.W., Determination of errors in individual discharge measurements. US Geological Survey, Open File Report 92-144, Nacross, GA, 1992.
  - [27] Herschy, R.W., 1994. The analysis of uncertainties in the stage-discharge relation. *Flow. Meas. Instrum.*, 5(3) Oxford: Butterworth-Heinemann, 188–190.
  - [28] R.W. Herschy, Accuracy, in *hydrometric measurements*, in *Hydrometry Principles and Practices*, second ed., Wiley, Chichester, UK, 1998.